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A note on fake surfaces and universal covers

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Abstract

In this paper, we consider compact fake surfaces X with the property that each component of the subgraph $\Gamma \subset X$ of triple edges contains at most one point whose link in X is homeomorphic to the 1-skeleton of a tetrahedron (type III). Assuming the subgroups $\Lambda_i \subseteq \pi_1(X)$ generated by the loops in Γ at the points v_i of type III are of a certain kind, an application of F.F. Lasheras [Proc. Amer. Math. Soc. 128 (2000) 893–902; J. Pure Appl. Algebra 151 (2) (2000) 163–172] leads us to finding a compact polyhedron K with $\pi_1(K) \cong \pi_1(X)$ and whose universal cover $\widetilde{K}$ has the proper homotopy type of a 3-manifold. This result extends the work in F.F. Lasheras [J. Pure Appl. Algebra 151 (2) (2000) 163–172] about a question on finitely presented groups.

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1. Introduction

It is well known that if G is a finitely generated group then its first cohomology group with $\mathbb{Z}G$-coefficients $H^1(G;\mathbb{Z}G)$ is free Abelian and it counts the number of ends of G, namely, this number equals $1 + \operatorname{rank}(H^1(G;\mathbb{Z}G))$ [1]. If G is finitely presented then its second cohomology $H^2(G;\mathbb{Z}G)$ has been shown to be torsion free (see [2]) and the freeness of this Abelian group is related to the *semistability of G at infinity*, i.e., whether or not any two proper rays in the universal cover $\widetilde{X}$ defining the same end are properly homotopic, where X is a finite 2-complex with $\pi_1(X) \cong G$. Geoghegan and Mihalik [2] showed that if G is semistable at infinity then $H^2(G;\mathbb{Z}G)$ is free Abelian.

The question of whether or not this second cohomology group is free Abelian for all finitely presented groups is well-known. We consider the following stronger question, which was formulated in [5] for any finitely presented group G:

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(Q) *Does there exist a finite* 2*-complex* K *having* G *as fundamental group and whose universal cover* $\widetilde{K}$ *has the proper homotopy type of a* 3*-manifold* M (*with boundary*)?

Indeed, an affirmative answer to (Q) for a group G would give us the freeness of the group $H^2(G;\mathbb{Z}G)$, since the cohomology group with compact support $H_c^2(\widetilde{K};\mathbb{Z})$ is isomorphic to a direct sum $H^2(G;\mathbb{Z}G) \oplus (\textit{free Abelian})$ (see [2]), and Lefschetz duality tells us that $H_c^2(\widetilde{K};\mathbb{Z}) \cong H_1(M,\partial M;\mathbb{Z})$ is free Abelian, since simply connected manifolds are orientable.

It is important to note that every finitely presented group can be realized as the fundamental group of a compact fake surface (see Section 2). In [5] it was shown that we have an affirmative answer to (Q) in the case G is the fundamental group of a compact fake surface with no vertices of type III, i.e., the link of any vertex is not homeomorphic to the 1-skeleton of a tetrahedron.

In this paper, we continue in the line of [5] dealing with compact fake surfaces X with the property that each connected component of the graph $\Gamma \subset X$ of all triple edges contains at most one vertex of type III. In this situation, let $\gamma_{i,0}, \gamma_{i,1} \in \pi_1(X)$ denote the homotopy classes of the two simple closed curves in Γ containing vertices $v_i \in X$ of type III, and let $\Lambda_i \subseteq \pi_1(X)$ be the subgroup generated by $\gamma_{i,0}$ and $\gamma_{i,1}$. Let us recall that a T-bundle is a bundle whose fiber T consists of three segments sharing a vertex. With this notation, we have:

Theorem 1.1. *Let* X *be a compact fake surface as above. Suppose that each of the subgroups* $\Lambda_i \subseteq \pi_1(X)$ *is isomorphic to either* $\langle\gamma_{i,0}\rangle * \langle\gamma_{i,1}\rangle$ *or* $\langle\gamma_{i,0}\rangle \oplus \langle\gamma_{i,1}\rangle$, *and that every* $\gamma_{i,j}$ *of* (*finite*) *odd order has a* T*-bundle over it in* X *not containing a Möbius band. Then, there is a compact* 2*-dimensional polyhedron* $K \supset X$ *with* $\pi_1(K) \cong \pi_1(X)$ *and whose universal cover* $\widetilde{K}$ *is proper homotopy equivalent to a* 3*-manifold.*

Thus, we get an affirmative answer to (Q) for all those groups G which are the fundamental group of a compact fake surface as in Theorem 1.1.

Under certain hypothesis, we will also allow a relation of the form $\gamma_{i,0}^m = \gamma_{i,1}^n$ in the subgroups $\Lambda_i \subseteq \pi_1(X)$ (see Remark 3.2).

2. Fake surfaces

Let us recall some definitions and results from [4].

Definition 2.1. Let X be a 2-dimensional locally finite simplicial complex, and assume the link $lk(v,X)$ is planar, for every vertex v of X. Let (v,w) be a 1-simplex of X. Given an embedding $\phi_v : lk(v,X) \to \mathbb{R}^2$, we denote by $\theta_{\phi_v}(w)$ the cyclic ordering determined by ϕ_v on $lk((v,w),X)$ as we go around $\phi_v(w)$ following the orientation in $\mathbb{R}^2$. Note that if the cardinality $|lk((v,w),X)|$ is $\leqslant 2$, then there is only one cyclic ordering $\theta_{\phi_v}(w)$.

We denote by $\Gamma \subset X$ the graph consisting of all the 1-simplexes of X which are of order > 2 (i.e., those which are the face of at least three 2-simplexes of X). Consider the cochain complex of Γ over $\mathbb{Z}_2$

$$0 \to C^0(\Gamma; \mathbb{Z}_2) \xrightarrow{\delta} C^1(\Gamma; \mathbb{Z}_2) \to 0.$$

Given a family $\Phi = \{\phi_v : lk(v, X) \to \mathbb{R}^2,\ v \in X^0\}$ of embeddings, we can associate to it a cochain (cocycle) $\omega_\Phi = \sum_{\sigma \subset \Gamma} \omega_\Phi(\sigma) \cdot \sigma \in C^1(\Gamma; \mathbb{Z}_2)$ where $\omega_\Phi(\sigma) = 0$ if $\theta_{\phi_{o(\sigma)}}(t(\sigma))$ and $\theta_{\phi_{t(\sigma)}}(o(\sigma))$ are opposite, and $\omega_\Phi(\sigma) = 1$, otherwise. Here $o(\sigma)$ and $t(\sigma)$ are the vertices of σ. By extension, we define $\omega_\Phi(\sigma) = 0$ for every 1-simplex σ of order $\leqslant 2$. Given an edge path γ in Γ, we will write $\omega_\Phi | \gamma \neq 0$ if there is a 1-simplex $\sigma \subset \gamma$ so that $\omega_\Phi(\sigma) \neq 0$.

This cochain is key to detecting the thickenability of a complex to a 3-manifold. More specifically, we say that a simplicial complex X "thickens" to a polyhedron Y if Y admits a CW-structure containing, as a subcomplex, a copy of a subdivision of X onto which Y collapses. The following theorem is proved in [4].

Theorem 2.2. *Let X be a 2-dimensional connected locally finite simplicial complex. Then, X thickens to an orientable 3-manifold if and only if*
(i) *$lk(v, X)$ is planar, for every vertex v of X, and*
(ii) *there exists a family of embeddings $\Phi = \{\phi_v : lk(v, X) \to \mathbb{R}^2,\ v \in X^0\}$ so that the associated cochain ω_Φ is trivial.*

Next, we recall the concept of a "fake surface" which was introduced by Ikeda [3] in 1971 while studying a conjecture of Zeemann. Although the term "fake surface" traditionally applies to a class of compact polyhedra, we will allow non-compact fake surfaces. Explicitly,

Definition 2.3. A 2-dimensional locally compact polyhedron X is called a (*closed*) *fake surface* if each point in X has a neighborhood of one of the following p.l. types. See Fig. 1.

In particular, $lk(x, X)$ is planar for every point x of X.

Given a fake surface X and a triangulation K of X, we denote by $\Gamma \subset X$ the 1-dimensional subpolyhedron determined by the subgraph of K consisting of all the 1-simplexes of order 3, i.e., those which are the face of three 2-simplexes. Observe that Γ is independent

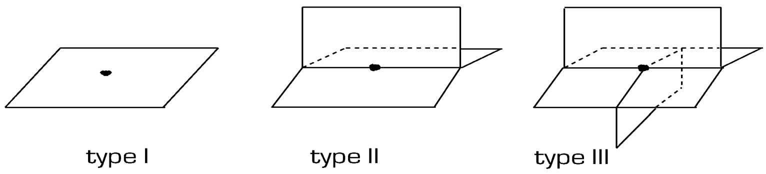

Fig. 1.

of the triangulation K of X, and it consists of all those points of X of types II and III. We also have that every point of type III is a vertex of the triangulation.

Remark 2.4. It is worth mentioning that every compact connected 2-dimensional polyhedron is simple homotopy equivalent to a compact fake surface (see [7]). This way, we can think of a finitely presented group G in terms of (compact) fake surfaces having G as fundamental group.

The following theorem is proved in [4,6].

Theorem 2.5. *Let X be a fake surface, and $\Gamma \subset X$ be as above. There exists a well defined cohomology class $\xi_X \in H^1(\Gamma; \mathbb{Z}_2)$ with the property that $\xi_X = 0$ if and only if X thickens to an orientable 3-manifold. In fact, ξ_X is defined as $[\omega_\Phi]$, for any family Φ of embeddings.*

From now on, we will consider triangulated compact fake surfaces X such that each component of $\Gamma \subset X$ contains at most one vertex of type III. Notice that, in this situation, if J is a connected component of Γ, then J is homeomorphic to either the wedge $S^1 \vee S^1$ or the circle S^1, depending on whether J contains a vertex of type III or not (respectively), since X is compact and every vertex of Γ has either valence 2 (vertex of type II) in Γ or valence 4 (vertex of type III).

3. Universal covers and 3-manifolds

The purpose of this section is to prove Theorem 1.1. We will keep the notation given in the introductory section.

Proof of Theorem 1.1. We consider a triangulation on X, leading us to a finite 2-dimensional simplicial complex, which will also be denoted by X. This complex has the property that each component of the subcomplex $\Gamma \subset X$, consisting of all the 1-simplexes of order 3, has at most one vertex of type III. Let $\{v_i,\ i \in I\}$ be the (finite) set of vertices of type III of X, and let $J_i \subseteq \Gamma$ denote the component of Γ containing v_i. We keep denoting by $\gamma_{i,0}, \gamma_{i,1} \in \pi_1(X)$ the homotopy classes of the two simple closed curves in $J_i \cong S^1 \vee S^1$, and by $\Lambda_i = \langle \gamma_{i,0}, \gamma_{i,1} \rangle \subseteq \pi_1(X)$ the subgroup generated by them. By abuse of notation, the closed curves in J_i representing the corresponding generators of Λ_i will also be denoted by $\gamma_{i,0}$ and $\gamma_{i,1}$.

Let $\Phi = \{\phi_v : lk(v, X) \to \mathbb{R}^2, v \in X^0\}$ be a family of embeddings on X. Given this family Φ, we can always find a cochain $\omega \in C^1(\Gamma; \mathbb{Z}_2)$ cohomologous to ω_Φ with the property that if $C \subset \Gamma$ is a simple closed curve, then $\omega|C \neq 0$ only if $\omega_\Phi|C \neq 0$, and in that case there is only one 1-simplex $\sigma \subset C$ for which $\omega(\sigma) \neq 0$. Note that $\omega = \omega_{\Phi'}$ for some family Φ' of embeddings on X (see [4]). We replace Φ with Φ', and denote this new family by Φ again. Let $p : \widetilde{X} \to X$ be the universal covering map, and let $\widetilde{\Phi}$ be the family of embeddings on $\widetilde{X}$ obtained as a lift of the family Φ to the universal cover.

The proof of [5, Theorem 1.2] shows that there is a finite 2-complex $K \supset X$ with $\pi_1(K) \cong \pi_1(X)$, and whose universal cover $\widetilde{K}$ is proper homotopy equivalent to a locally finite 2-complex W with planar links, for which there is a family $\Psi = \{\psi_w : lk(w, W) \to \mathbb{R}^2,\ w \in W^0\}$ of embeddings such that $\omega_\Psi(\sigma) \neq 0$ only for those 1-simplexes σ in $p^{-1}(\bigcup_{i \in I} J_i) \subseteq \widetilde{X}$ with $\omega_{\widetilde{\Phi}}(\sigma) \neq 0$. The complex K is constructed as follows. Set $\widetilde{\Gamma} = p^{-1}(\Gamma) \subset \widetilde{X}$, and let $L_j \subset \widetilde{\Gamma}$ be a compact component not containing vertices of type III, with the property that it separates a regular neighborhood $\widetilde{N}_j$ of it in $\widetilde{X}$ into two components. Let $\widetilde{S}_1, \widetilde{S}_2$ denote the boundary components of $\widetilde{N}_j$. We build K from X by attaching 2-cells along $S_k = p(\widetilde{S}_k),\ k = 1, 2$, via attaching maps of degrees n_k, where n_k is the order of the element of $\pi_1(X)$ determined by S_k. The 2-complex W, proper homotopy equivalent to $\widetilde{K}$, may be seen as the complex obtained from $\widetilde{K}$ by replacing the regular neighborhoods $\widetilde{N}_j$ with cylinders $S^1 \times I$. Thus, $\mathrm{cl}(\widetilde{X} - \bigcup_j \widetilde{N}_j)$ is a subcomplex of W. Let us prove the following lemma.

Lemma 3.1. *Suppose that each of the subgroups $\Lambda_i \subseteq \pi_1(X)$ is isomorphic to either $\langle\gamma_{i,0}\rangle * \langle\gamma_{i,1}\rangle$ or $\langle\gamma_{i,0}\rangle \oplus \langle\gamma_{i,1}\rangle$, and the family Φ of embeddings on X satisfies the condition that $\omega_\Phi|\gamma_{i,j} \neq 0$ implies $\gamma_{i,j}$ is not of (finite) odd order. Then, the 2-dimensional complex W described above thickens to a 3-manifold.*

Proof. Notice that if $w \in p^{-1}(\bigcup_{i \in I} J_i) \subset \mathrm{cl}(\widetilde{X} - \bigcup_j \widetilde{N}_j)$ is a vertex, them $\psi_w = \phi_w \in \widetilde{\Phi}$, by construction of Ψ. We are going to modify this family Ψ of embeddings on W by changing the embeddings ψ_w of some of the vertices of $p^{-1}(\bigcup_{i \in I} J_i)$, so as to obtain a new family Ψ' on W for which $\omega_{\Psi'}$ is clearly a coboundary, proving that W thickens to a 3-manifold, by Theorem 2.5. For this, we choose a vertex v_i of type III of X with $\omega_\Phi|\gamma_{i,0} \cup \gamma_{i,1} \neq 0$. In order to simplify notation, we will denote by γ_0, γ_1 the elements $\gamma_{i,0}, \gamma_{i,1}$. Recall that Φ (and hence $\widetilde{\Phi}$) has been chosen so that if $\omega_\Phi|\gamma_j \neq 0$, then there is exactly one 1-simplex σ with $\omega_{\widetilde{\Phi}}(\sigma) \neq 0$ on every lift to $\widetilde{X}$ of the loop γ_j. We may further assume that those simplexes σ with $\omega_{\widetilde{\Phi}}(\sigma) \neq 0$ do not contain vertices of type III. Observe that each component Σ of $p^{-1}(\gamma_0 \cup \gamma_1) \subset W$ is a copy of the Cayley graph of the group Λ_i, with $p^{-1}(v_i) \cap \Sigma$ as its set of vertices.

By hypothesis, the corresponding subgroup $\Lambda_i \subseteq \pi_1(X)$ is isomorphic to either $\langle\gamma_0\rangle * \langle\gamma_1\rangle$ or $\langle\gamma_0\rangle \oplus \langle\gamma_1\rangle$. We consider the following cases.

Case 1. $\Lambda_i \cong \langle\gamma_0\rangle * \langle\gamma_1\rangle$.

Case 1.1. γ_0, γ_1 are both of infinite order. In this case, each component Σ of $p^{-1}(\gamma_0 \cup \gamma_1)$ is the “snowflake”. We can take new embeddings ψ_w for the link of each vertex w on each of those snowflakes so that ω_Ψ vanishes on them. For this, we pick a vertex on each snowflake Σ and start going away from that vertex choosing embeddings for the link of the vertices we encounter on the corresponding snowflake, so as to get $\omega_\Psi(\sigma) = 0$ for every 1-simplex $\sigma \subset \Sigma$.

Next, we suppose that one of the generators of Λ_i is of finite order. By symmetry, we may assume that γ_0 is of finite order. We consider the following cases, depending on $\omega_\Phi|\gamma_0$.

Case 1.2. $\omega_\Phi|\gamma_0 \neq 0$. As before, we concentrate on each component $\Sigma \subset p^{-1}(\gamma_0 \cup \gamma_1)$. Fix a vertex $w_0 \in p^{-1}(v_i) \cap \Sigma$. We extend $\{w_0\}$ to a subset $V \subset p^{-1}(v_i) \cap \Sigma$ of vertices of type III, according to the following rules: (i) if $w \in V$, then $w \cdot \gamma_0^2 \in V$ and $w \cdot \gamma_0 \notin V$; (ii) for each $w \in V$, we have $w \cdot \gamma_1 \in V$ only if $\omega_\Phi|\gamma_1 = 0$, and $w \cdot (\gamma_1\gamma_0) \in V$, otherwise. Note that there is an even number of vertices of type III on each component of $p^{-1}(\gamma_0)$, since γ_0 is of even order, by hypothesis. Next, we replace the embedding ψ_w with $\psi'_w = h \circ \psi_w$ for every vertex $w \in V$, where h is an orientation-reversing homeomorphism of $\mathbb{R}^2$. This way, we get a family Ψ' of embeddings on W so that there are two 1-simplexes σ with $\omega_{\Psi'}(\sigma) \neq 0$ on every lift of γ_0, and there are either zero or two 1-simplexes σ with $\omega_{\Psi'}(\sigma) \neq 0$ on every lift of γ_1 (depending on whether that lift contains vertices of V). Thus, it is clear that $\omega_{\Psi'}$ is a coboundary on each component $\Sigma \subset p^{-1}(\gamma_0 \cup \gamma_1)$.

Case 1.3. $\omega_\Phi|\gamma_0 = 0$ and γ_1 is of infinite order (with $\omega_\Phi|\gamma_1 \neq 0$). Fix a vertex $w_0 \in p^{-1}(v_i)$ on each component Σ of $p^{-1}(\gamma_0 \cup \gamma_1)$. We extend $\{w_0\}$ to a subset $V \subset p^{-1}(v_i) \cap \Sigma$ satisfying the following conditions: (i) if $w \in V$, then $w \cdot \gamma_0 \in V$; (ii) for each $w \in V$, we have $w \cdot \gamma_1^2 \in V$ and $w \cdot \gamma_1 \notin V$. For every vertex $w \in V$, we replace ψ_w with $\psi'_w = h \circ \psi_w$, as before. Thus, we obtain a family Ψ' of embeddings so that there are two 1-simplexes σ with $\omega_{\Psi'}(\sigma) \neq 0$ on every lift of γ_1, and there are either zero or two 1-simplexes σ with $\omega_{\Psi'}(\sigma) \neq 0$ on every lift of γ_0. Again, it is clear that $\omega_{\Psi'}$ is a coboundary on each component Σ.

Case 2. $\Lambda_i \cong \langle\gamma_0\rangle \oplus \langle\gamma_1\rangle$. This case is the Abelian version of case 1. Since $\omega_\Phi|\gamma_0 \cup \gamma_1 \neq 0$, by hypothesis, there must be a generator of Λ_i, say γ_0, for which $\omega_\Phi|\gamma_0 \neq 0$. We now proceed just as in case 1.2 above, even if γ_0 is not of finite order, obtaining a family Ψ' of embeddings on W so that $\omega_{\Psi'}$ is clearly a coboundary on each component of $p^{-1}(\gamma_0 \cup \gamma_1)$. □

We continue now with the proof of Theorem 1.1. Let us recall from [3,8] that given a fake surface Y, a simple closed curve $S \subset Y$ of triple edges and a regular neighborhood N of S in Y, there is embedded in N a (uniquely determined) T-bundle over S, obtained by gluing the two ends of $T \times I$ via a permutation of the three segments of T. If we number these three segments, this T-bundle is of one of the following types: the one induced by (i) the identity permutation; (ii) the 3-cycle (123); and (iii) the 2-cycle (12). Let $S \subset Y$ be one of such simple closed curves determining an element of odd order r in $\pi_1(Y)$, and suppose S has a T-bundle Q over it in Y of type (iii). We observe that each component $\widetilde{S}_j$ of $p^{-1}(S) \subset \widetilde{Y}$ also has a T-bundle $\widetilde{Q}_j$ in the universal cover $\widetilde{Y}$ of type (iii). The diagram below roughly describes $p|_{\widetilde{Q}_j}$, which is an r-sheeted covering map from one of such T-bundles $\widetilde{Q}_j$ to Q (the centerline represents $\widetilde{S}_j$). See Fig. 2.

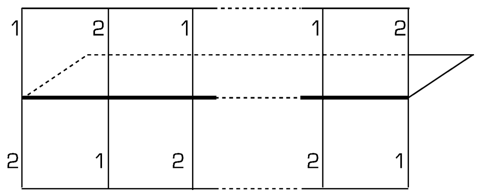

Fig. 2.

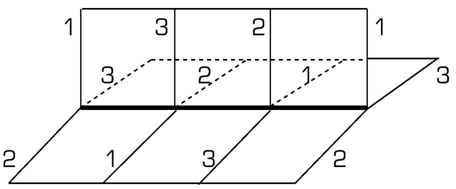

Fig. 3.

In this case, $\widetilde{Y}$ does not thicken to any orientable 3-manifold M, since otherwise each component $\widetilde{S}_j$ would have a solid Klein bottle as a regular neighborhood in M (see also [6]).

In the context of Theorem 1.1, if $\omega_\Phi|\gamma_{i,j} \neq 0$ for some $\gamma_{i,j} \in \pi_1(X)$ of odd order, then it is not guaranteed that the 2-complex W itself thickens to a 3-manifold, unless $\gamma_{i,j}$ has a T-bundle over it in X of type (i) or (ii). In the latter case, we observe that, since the cyclic permutation (123) has order 3, each component $\tilde{\gamma}_{i,j} \subset p^{-1}(\gamma_{i,j})$ has a T-bundle in W of type (i) or (ii), depending on whether the order of $\gamma_{i,j}$ in $\pi_1(X)$ is a multiple of 3 or not, respectively. See the diagram in Fig. 3 describing the restriction of p from a (trivial) T-bundle over $\tilde{\gamma}_{i,j}$ to a T-bundle over $\gamma_{i,j}$ (of type (ii)), in the case order$(\gamma_{i,j}) = 3$ (the centerline represents $\tilde{\gamma}_{i,j}$).

In either case, it is an easy exercise to check that we can take embeddings ψ'_w for the vertices $w \in \tilde{\gamma}_{i.j}$ so that for every 1-simplex $\sigma = (v, w) \subset \tilde{\gamma}_{i,j}$, the cyclic orderings $\theta_{\psi'(v)}(w)$ and $\theta_{\psi'(w)}(v)$ are opposite (see Section 2). Thus, $\omega_{\Psi'}|\tilde{\gamma}_{i,j} = 0$ for every such component $\tilde{\gamma}_{i,j} \subset W$. $\square$

Remark 3.2. Suppose we have a vertex $v_i \in X$ of type III with $\omega_\Phi|\gamma_0 \cup \gamma_1 \neq 0$, for which the corresponding subgroup $\Lambda_i \subseteq \pi_1(X)$ has the form $\langle \gamma_0, \gamma_1; \gamma_0^m \gamma_1^{-n} \rangle$. If one of the following statements holds:

(a) m, n are both odd, γ_0, γ_1 are not of odd order and $\omega_\Phi|\gamma_0 \neq 0 \neq \omega_\Phi|\gamma_1$;

(b) n is even, γ_1 is not of odd order and $\omega_\Phi|\gamma_0 = 0 \neq \omega_\Phi|\gamma_1$;

then, we can get a family Ψ' of embeddings on the 2-complex W, described above, so that the corresponding cochain $\omega_{\Psi'}$ is clearly a coboundary on each component of $p^{-1}(\gamma_0 \cup \gamma_1)$.

Proof. Let Σ be a component of $p^{-1}(\gamma_0 \cup \gamma_1)$. Assume (a) holds. We take a subset $V \subset p^{-1}(v_i) \cap \Sigma$ of vertices of type III satisfying the property that if $w \in V$, then

$$w \cdot \gamma_j^2 \in V \quad \text{and} \quad w \cdot \gamma_j \notin V, \quad j = 0, 1.$$

Then, for each $w \in V$, we replace the embedding ψ_w with $\psi'_w = h \circ \psi_w$, as in the proof of Theorem 1.1. On the other hand, if (b) holds, then we take a subset $V \subset p^{-1}(v_i) \cap \Sigma$ satisfying the conditions: (i) if $w \in V$, then $w \cdot \gamma_0 \in V$; (ii) for each $w \in V$, $w \cdot \gamma_1^2 \in V$ and $w \cdot \gamma_1 \notin V$. Again, we replace ψ_w with $\psi'_w = h \circ \psi_w$, for every $w \in V$. In either case, one can check that there are either zero or two 1-simplexes σ with $\omega_{\Psi'}(\sigma) \neq 0$ on every lift of γ_j $(j = 0, 1)$. $\square$

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